Calculus III Midterm Exam

March 4, 2020

Don't forget to write down clearly your

Name: ______________________ **Net ID:** ______________________

Instructions.

- The exam book contains 6 basic problems, worth 100 points.
- The total time for the exam is 75 minutes.
- No books, notes or calculators are allowed.
- Read the following story before opening the exam book!

  A mathematician, a physicist, and an engineer were traveling through Scotland when they saw a black sheep through the window of the train.

  "Aha," says the engineer, "I see that Scottish sheep are black."

  "Hmm," says the physicist, "You mean that some Scottish sheep are black."

  "No," says the mathematician, "All we know is that there is at least one sheep in Scotland, and that at least one side of that one sheep is black!"

  So, please be precise with your answers just as the mathematician in the story is!

- Good luck with the exam!

FOR GRADERS ONLY

Problem Number	Points
1	
2	
3	
4	
5	
6	
Total Points	

1. Right triangle (15 points) Show that the triangle with vertices $A = (1, 1, 1)$, $B = (2, 3, 4)$ and $C = (3, 0, 1)$ is a right triangle.

2. Space curve (20 points) Let $\mathbf{r}(t)$ be the space curve

$$\mathbf{r}(t) = (\cos(2\pi t), \sin(2\pi t), \ln \sin(2\pi t))$$

where $0 \leq t \leq \frac{1}{2}$.

(1) Find the unit tangent vector of $\mathbf{r}(t)$ at $t = \frac{1}{4}$.

(2) Find the total length of the space curve $\mathbf{r}(t)$ above.

3. Tangent plane (15 points) Consider the function $f(x, y, z) = x^2 + 2y^2 + z - 6xy$ on $\mathbb{R}^3$.

(1) In what unit direction is the function $f(x, y, z)$ increasing the fastest at the point $P = (1, 1, 3)$?

(2) Find the equation of tangent plane of the level surface $f(x, y, z) = \mathrm{const}$ passing through the point $P = (1, 1, 3)$.

4. Volume of a solid (15 points) Find the volume of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 2)$.

5. Double integral (15 points) Let D be the region between the circles $(x-2)^2+y^2=4$ and $(x-4)^2+y^2=16$. Evaluate the integral

$$\iint_D x dA.$$

6. Extreme values. (20 points) Find the absolute maximum and minmum of the function $g(x, y) = xy + y^2 + x^2$ on the disk $D = \{(x, y)|x^2 + y^2 \leq 4\}$.