

Graphing Functions

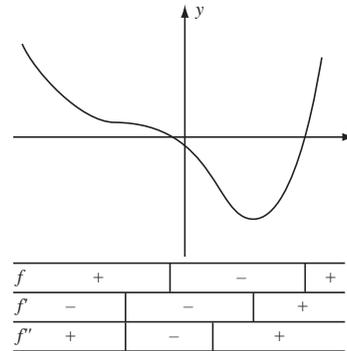
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Objective

To investigate what we can say about the graph of a function f given where f and its first and second derivatives are positive, negative, and zero.

Narrative

To record where a function f and its first and second derivatives f' and f'' , respectively, are positive, negative, and zero, we may use three “recording strips” below the graph of f as illustrated in the figure at the right. Study this figure carefully. Note how the vertical bars are drawn where f , f' , and f'' are 0, and the spaces between the vertical bars are labelled + or – depending on the sign of f , f' , and f'' over those intervals. (Recall that the graph of f is:



- a) increasing if $f'(x) > 0$, b) decreasing if $f'(x) < 0$,
- c) concave up if $f''(x) > 0$, d) concave down if $f''(x) < 0$.)

In this project we will use this mechanism to investigate the graphs of functions. To reduce the amount of code you need to enter, a function `drawgrid` has been created for your convenience. The command “`drawgrid`” draws a coordinate grid and recording strips in the current figure window.

To use `drawgrid`, you must have it saved on your computer and a path set to it in MATLAB so that it is accessible. Since the way this is done depends on the computer you are using, we make minimal specific comments here. On the other hand, our use of `drawgrid` illustrates several important points: First, it illustrates that when a large amount of code is to be typed more than once (to be “reused”), it is very useful to have that code saved as a function that can be easily called. Second, it illustrates a second way user-defined functions can be implemented in MATLAB — in a MATLAB m-file. An m-file is a text file that can be created either from within MATLAB, or from within a text editor. More will be said later about the proper form of a function defined by an m-file.

Tasks

1. Type the commands below into MATLAB. They draw the graph of $f(x) = 4x/(1+x^2)$, and three recording strips below the graph of f .

```
>> % Your name, today's date
>> % Graphing Functions
>> clear all, close all
>> syms x
>> % Task 1
>> f = inline('4*x/(1+x^2)')
>> figure(1)
>> hold on
>> drawgrid
>> ezplot(f, [-6.0,6.0,-6.0,2.5])
>> hold off
```

2. a) Continue by typing the commands below into MATLAB. They find where $f(x) = (x^4+4x^3-16x-16)/10$ and its first and second derivatives are zero; this information will be used later in this project.

```

>> % Task 2a
>> f = inline('(x^4+4*x^3-16*x-16)/10')
>> solve(f(x),x)
>> f1 = diff(f(x),x)
>> solve(f1,x)
>> f2 = diff(f1,x)
>> solve(f2,x)

```

b) Type the commands below into MATLAB. They produce an empty graph and three recording strips.

```

>> % Task 2b
>> figure(2)
>> hold on
>> drawgrid
>> title('(x^4+4*x^3-16*x-16)/10')
>> hold off

```

At this time make hard-copies of MATLAB's figure windows. (But don't shut down MATLAB yet! It will be very helpful in what is to come!) Then:

3. Fill in the recording strips below the graphic you produced in Task 1 with:

- a) the labels f , f' , and f'' , and
- b) +'s and -'s using the graph of f as a guide.

4. Fill in the recording strips on the graphic you produced in Task 2(b) with:

- a) the labels f , f' , and f'' ,
- b) vertical bars corresponding to the zeros of f , f' , and f'' using the information you computed in Task 2(a), and
- c) +'s and -'s. (You will need to evaluate f , f' , and f'' between their respective zeroes to determine where they are positive and negative. This is where MATLAB can still be of use to you: using it will make your evaluations easier. To evaluate `f1` at $x = 0.5$, for example, you just need to type “`subs(f1,x,0.5)`” into MATLAB's Command Window.)
- d) Use the information you've recorded to sketch the graph of f in the space provided.

At this time, make a hard copy of MATLAB's Command Window. Your hard copies of MATLAB's Command Window and figure windows will be your lab report for this project. Staple these hard copies together, the Command Window on top, and your figure windows underneath.

Comments

To graph an arbitrary function f we need to determine not only where f and its first and second derivatives f' and f'' are positive, negative, and zero, but also where they do not exist. We did not have to consider where they do not exist in this project since f was a polynomial, and hence it and its first and second derivatives f' and f'' exist everywhere.

In general, however, the points where:

1. $f(x) = 0$ are the x -intercepts of the graph of f ,
2. $f(x)$ does not exist are the x -intercepts of the vertical asymptotes of the graph of f ,
3. $f'(x) = 0$ are critical points of the graph of f ,

4. $f'(x)$ does not exist are the x -intercepts of the vertical tangents to the graph of f ,
5. $f''(x) = 0$ and $f''(x)$ do not exist are the x -intercepts of the possible inflection points of the graph of f .

For a function f for which $f(x)$, $f'(x)$, and/or $f''(x)$ did not exist for certain values of x , we would draw and label vertical bars in our recording strips with $\pm\infty$.